Distributed method for Economic Dispatch Problem with a battery system and a variable fuel price

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***Abstract*—The distribution of the required load across all generators in power networks has given rise to methods solving the Economic Dispatch Problem (EDP). The main goal of the method is to find such a power distribution that the resulting price per unit of energy is minimal. Due to dynamic changes in the structure of the energy network is there currently a shift away from centralized management towards decentralized methods. In this paper, a gradient-based distributed algorithm is introduced. In proposed framework, models for Battery Energy Storage System (BESS) and variable fuel price are used. The BESS model allows energy to be stored or fed back into the grid, which can affect the final price per unit of energy. Dynamical behaviour of BESS effects final price per energy of generator units. This is equally related to the price of fuel, which can represent a change in the price of energy from the given source - the power plant. The described algorithm is subsequently validated on three examples.**

***Index Terms*—Battery system, BESS, distributed approach, Economic Dispatch Problem, fuel price, gradient method, multi-agent, power networks, price, smart grid.**

I. Introduction

Energy networks belong to the critical infrastructure of today's modern society. They connect electricity producers and consumers, which include both households and industrial companies. The energy system must be managed and controlled at many levels, from individual generators to entire areas, in order to cover the required load among the entire network. The Economic Dispatch Problem (EDP) deals with distributing the required load across all online generators present in the network. The primary goal is to get the lowest possible final price per unit energy as possible [1, 2, 3].

It is thus an optimization task, whose input is the considered network topology. Network topology means the information about the connected generators but also information about the value of the required amount of produced energy. Mathematical models and statistical methods are used to predict this value over the considered time horizon [4]. The unit commitment problem, which deals with the selection of generators to be active, is also related to the EDP topic[5].

There are many ways to solve the EDP. An overview of methods utilized around 1990 is presented in [3]. This typically includes centralized solution methods. Concretely, it can be the Lambda iteration method or genetic programming [6, 7]. One can also encounter approaches using particle swarm optimization methods or evolutionary programming [8, 9, 10]. Over time, however,there has been a move away from classical methods towards a decentralized solutions, primarily due to the development of multi-agent systems theory and technological progress [11, 2]. A large number of consensus-based algorithms can be implemented in a distributed solution [12, 13, 14]. In recent years, there has also been a need to include the representation of renewable sources in the EDP which is mainly due to the step aside from conventional fossil energy sources in order to reduce mankind's air emissions [15, 16]. These efforts are leading to developments in solar, wind and other renewable energy-based power plants. Renewable energy resources are often supplemented by BESS [17, 18, 19]. Thus, the BESS can take energy from the grid if, for example, it is cheap. Or, on the contrary, feed it back into the grid if it is expensive or to cover peaks in energy demand [20, 18, 21]. The next logical extension was the representation of charging stations for electric cars. Electric cars are becoming increasingly common in real-world traffic and modern grid have to take them into account [22].

This paper extends the original algorithm presented in [12] with the representation of BESS and adds a variable fuel price. The price of a unit of electricity changes throughout the day and it is necessary to respond to it correctly [23, 24]. Typically, the price is scheduled using an auction-based system which can be one day ahead or in Europe, for example, in 15-minutes intervals [25, 26]. In determining the least-cost production plan, the variable fuel price is taken into account along with other costs such as fixed costs, start-up costs and no-load cost. In the case of BESS, it may be appropriate to buy energy and store it in batteries when the price is low and sell it again when the price rises above the considered limit. This consideration then depends on the specific business cases.

The paper is structured as follows. In the second chapter, the formulation of the problem with respect to battery systems is proposed together with a description of mathematical models. The third chapter presents the gradient-distributed algorithm. The fourth chapter is devoted to the verification of the algorithm on three prepared simulation examples. The following fifth chapter is dedicated to the possibilities of further development. And finally, the last chapter contains the conclusion and summary of the whole paper.

II. Problem formulation

A typical formulation of the Economic Dispatch Problem is as follows: [12, 1]:

$$\min_{x_i} \sum_{i=1}^{N} C_i(x_i).$$

Where C_i represents the cost function of the i-th generator. Parameter x_i denotes the power generation of the i-th generator and N the number of all generators,. In the case of considering the inclusion of BESS, the problem formulation is modified as follows [27, 18]:

$$\min\Big[\sum_{i=1}^{N_c} C_{c_i}(x_{c_i}) + \sum_{i=1}^{N_b} C_{b_i}(x_{b_i})\Big]. \quad (1)$$

Where N_c and N_b represent the number of classical[1] and battery systems respectively, x_{c_i} and x_{b_i} are the power generation of the corresponding i-th generator or battery system. C_{c_i} and C_{b_i} are the corresponding cost functions of the i-th facilities.

Note 1: The formulation in (1) can be further extended to include renewable sources [16]. However, due to the limited capacity, they are not considered in this paper.

For x_{c_i} and x_{b_i} the following applies:

$$\begin{aligned} \sum_{i=1}^{N_c} x_{c_i} \pm \sum_{i=1}^{N_b} x_{b_i} &= D + P_{\text{loss}}, \\ x_{c_i} \in X_{c_i} := [x_{c_i}^{min}, x_{c_i}^{max}], &\quad i = 1, ..., N_c, \\ x_{b_i} \in X_{b_i} := [x_{b_i}^{min}, x_{b_i}^{max}], &\quad i = 1, ..., N_b, \end{aligned} \quad (2)$$

where the symbol $\pm$ for BESS indicates that they can be charged and discharged [28]. This means they can supply stored power to the network or, conversely, draw energy from the grid and store it [20]. Sometimes it is possible to include losses on the power network line. These are referred to as P_{loss} and are added to the total required load D. Wire losses are usually quoted as 5% [29]. x_i^{min} and x_i^{max} are the lower and upper bounds for the i-th generator or BESS. The following formulation must be applied to cover the required load D [29, 28, 27, 18]:

$$\sum_{i=1}^{N_c} x_{c_i}^{min} \pm \sum_{i=1}^{N_b} x_{b_i}^{min} \leq D \leq \sum_{i=1}^{N_c} x_{c_i}^{max} \pm \sum_{i=1}^{N_b} x_{b_i}^{max},$$

to ensure solvability of (1). Furthermore, it is important to specify the conditions for discharging the battery:

$$\sum_{T_{d_i}} x_{b_i} \leq \sum_{T_{b_i}} x_{b_i}.$$

This can be interpreted as the power supplied in time T_d must not exceed total energy previously stored in time T_b for the i-th BESS. The generator price functions C_{c_i} can be expressed in different forms [12, 30, 18, 2]. The price function for classical generators $C_{c_i}(x_{c_i})$ will be considered in the quadratic form. Also, the convex price function is widely used in the EDP solutions [12, 24, 2]:

$$C_{c_i}(x_{c_i}) = \frac{(x_{c_i} - \alpha_{c_i})^2}{2\beta_{c_i}} + \gamma_{c_i}. \quad (3)$$

Parameters α, β and γ denotes the i-th generator parameters. They are listed by the producer or can be reconstruct by the power curve approximation utilizing a quadratic function. They are chosen as $\alpha_{c_i} \leq 0$, $\beta_{c_i} > 0$ and $\gamma_{c_i} \leq 0$ [2, 29].

[1] Classical generator refers to coal, oil, gas and nuclear resources.

A. Representation of the battery system

In this section, two models will be presented that can be used to represent the BESS. When the first will be simpler and the second more complex, with more adjustable parameters.

*1) **Simple model:*** This type of model does not consider any extra adjustable parameters. It only utilizes input power for its work, which has to be stored in batteries. The total energy stored in the BESS is then expressed by the formula [17]:

$$E_b = \sum_{i=1}^{T_b} x_{b_i} \cdot T_{b_i},$$

where T_b represents the total time period under considered. Next, x_{b_i} indicates the power accumulated in the i-th BESS. The variable x_{b_i} can be determined according to the following prescription:

$$x_{b_i} = \frac{\int_t x_{\text{input}}\, dt}{\int_t dt}. \quad (4)$$

Equation (4) denotes the energy stored in a particular i-th device. The input value x_{input} represents the energy taken from the rest of the grid which has to be stored in the i-th battery system.

The price function $C_{b_i}(x_{b_i})$ of the system modelled in this way can be considered in the following form:

$$C_{b_i}(x_{b_i}) = \alpha_{b_i} \cdot x_{b_i},$$

where the α_{b_i} is an optional parameter representing the price of a given installation, wear and tear over time and the total purchase price of the i-th BESS.

Note 2: It is clear from the above that this is a simple model, which may not always be suitable. It does not respect charge and discharge efficiencies of the BESS, as well as their maximum values. However, it can be extended for minimum C_{min} and maximum C_{max} values of stored energy.

$$C_{\text{min}} \leq E_b \leq C_{\text{max}}$$

*2) **More complex model:*** A more sophisticated model takes into account more adjustable parameters as well as more real-world constraints [30]. It is therefore a more appropriate choice if a more accurate representation is required.

First of all, it is a limitation of the maximum and minimum capacity, typically it can be, for example, 20% and 80% or 10% and 90%. This limitation is mainly considered due to the durability and efficiency of the battery systems. Another limitation represents the choice of the limit for charging and discharging the battery system. This can be noted as:

$$\begin{aligned} 0 \leq P^{ch}_{\text{BESS}}(t) \leq P^{ch_{\max}}_{\text{BESS}}, \\ 0 \leq P^{dis}_{\text{BESS}}(t) \leq P^{dis_{\max}}_{\text{BESS}}, \end{aligned} \tag{5}$$

where $P^{ch}_{\text{BESS}}(t)$ and $P^{dis}_{\text{BESS}}(t)$ represent the charge and discharge rates of the i-th battery system. Both values at time t are then limited by a maximum value if they are exceeded.

Another requirement is that the battery system cannot be charged and discharged at the same time. Therefore, only one of these operations is performed at a time. This can be expressed as:

$$P^{ch}_{\text{BESS}}(t) \cdot P^{dis}_{\text{BESS}}(t) = 0.$$

The next condition is the storage constraints of the BESS, which can be written as:

$$\text{SOC}_L \leq \text{SOC}(t) \leq \text{SOC}_U$$
$$\text{SOC}(t) = \frac{C_{\text{BESS}}(t)}{C^{\max}_{\text{BESS}}},$$

where $\text{SOC}(t)$ represents the state of the charge at time t. Parameters SOC_L and SOC_U denote the lower and upper capacity limits respectively. Parameter $C^{\max}_{\text{BESS}}$ symbolize maximum capacity of BESS and $C_{\text{BESS}}(t)$ stands for actual capacity of BESS in time t:

$$C_{\text{BESS}}(t) = C_{\text{ini}} + P_{\text{BESS}^{\text{ch}}(t)} \cdot \eta_{\text{ch}} - P_{\text{BESS}^{\text{dis}}(t)} \cdot \eta_{\text{dis}},$$

where term C_{ini} is the initial value of the BESS capacity. Next, parameters η_{ch} and η_{dis} denote the charging and discharging efficiency of the storage battery respectively.

The last part of the model is the considered price function for the BESS described in this way. If more than one battery system is considered, it will again be an expression of the sum of the price functions:

$$\sum_{i=1}^{N_b} C_{b_i}(t) = \sum_{i=1}^{N_b} \pi_{\text{BESS}_i} |P_{\text{BESS}_i^{\text{ch}}(t)} + P_{\text{BESS}_i^{\text{dis}}(t)}|,$$

where the parameter π_{BESS_i} represents the i-th BESS consumption coefficient. This parameter thus represents the wear and tear of the BESS during charging and discharging, i.e. it actually penalizes any use of the BESS.

*3) **Comparison of the two models**:* To compare the behaviour of both models, an example of charging and discharging a battery system will be considered. It will be charged from the initialization value C_{ini} to the maximum value $C_{\max}$ and then discharged back to the initialization value, i.e. the minimum value $C_{\min}$. The parameters for the Simple model presented in subsection II-A1 are shown in Table I.

The parameters considered for the More complex model described in subsection II-A2 are listed in Table II where SOC_L and SOC_U essentially denote $C_{\min}$ and $C_{\max}$, respectively.

TABLE I: Parameters for the simple model

Parameters	Value
$x_{\text{input}} -$ charging [MWh]	10
$x_{\text{input}} -$ discharging [MWh]	10
C_{ini} [MW]	10
$C_{\min}$ [MW]	10
$C_{\max}$ [MW]	60
α_{b_i}	5

TABLE II: Parameters for the complex model

Parameters	Value
$P^{ch_{\max}}_{\text{BESS}}$ [MWh]	10
$P^{dis_{\max}}_{\text{BESS}}$ [MWh]	10
C_{ini} [MW]	10
SOC_L [MW]	10
SOC_U [MW]	60
η_{ch}	0.83
η_{dis}	0.83
π_{BESS} [\$/kWh]	0.1

For the sake of simplicity the charging P^{ch}_{BESS} and discharging P^{dis}_{BESS} will be considered throughout the simulation as their maximum value $P^{ch_{\max}}_{\text{BESS}}$ and $P^{dis_{\max}}_{\text{BESS}}$, respectively. At this point it is also good to mention that the simulation will be considered in seconds and therefore it is necessary to convert MWh units to MWs. MWh represents a unit per hour, so for the conversion we divide this value by 60 to get MWmin and then again by 60 to get MWs. The units MWh or kWh are more commonly used in the literature [30, 17, 18].

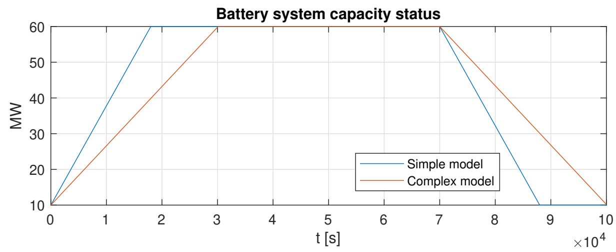


Fig. 1: Comparison between simple and complex models

The BESS charging and discharging simulations results for both models are shown in Figure 1.

Remark 1: From the results in Figure 1, it can be observed that both models give similar results. The more complex model differs in the rate of charge and discharge due to the consideration of the parameter for the efficiency of this operation - η_{ch} and η_{dis}, respectively. If a different charge value were considered, the results could be even more different.

III. Algorithm Definition

The modified version of the algorithm is based on the original version, utilizing Lagrangian dual method for EDP, which was described in [12]. It provides a distributed approach for EDP using a local multiplier λ estimation only with the help of local communication for each generator in the network.

The previous version did not consider the BESS and its representation and operator for a variable fuel price. It can be viewed as a variable price function, but its shape does not change, because the efficiency of the considered generator remains the same and only the fuel price changes. Its change can occur, for example, on the basis of information from the electricity exchange, when greater demand leads to a higher price of the resource. Or on the basis of economic facts, when the given commodity becomes more valuable, which manifests itself in longer time horizons. The main modified iterative algorithm consists of three equations that are calculated in each network agent:

$$\lambda_i(k+1) = \sum_{j=1}^{N} w_{ij}\lambda_j(k) - \begin{cases} \gamma_i(k)\frac{\nabla\Phi_i(\lambda_{c_i}(k))}{y_{ii}(k)}, \text{if } i \in N_c \\ \gamma_i(k)\frac{\nabla\Phi_i(\lambda_{b_i}(k))}{y_{ii}(k)}, \text{if } i \in N_b \end{cases} \tag{6}$$

$$x_i(k+1) = \begin{cases} \varphi_{c_i}\big(\nabla C_{c_i}^{-1}(\lambda_{c_i}(k+1)) \cdot \mu_{c_i}^{-1}(k)\big), & \text{if } i \in N_c \\ \varphi_{b_i}\Big(\Upsilon\big(\text{SOC}(t) \cdot \mu_{b_i}(k)\big)\Big), & \text{if } i \in N_b \end{cases} \tag{7}$$

$$y_i(k+1) = \sum_{j=1}^{N} w_{ij} y_j(k). \tag{8}$$

It can be noted that the first two equations can be calculated differently. It depends on whether it is an agent that represents a classical power plant (index c) or, on the contrary, a BESS (index b), which may have a different price function C_{b_i}. The parameter $\lambda_i(k+1)$ indicates the estimation of the optimal incremental cost by the i-th unit. $x_i(k+1)$ denotes the corresponding power generation and $y_i(k+1)$ represents the consensus variable. Parameters μ_{c_i} and μ_{b_i} marks the variable fuel price for the classical power plant and BESS, respectively, they are considered in percentage.

Nominal value for algorithm of current price is usually chosen as 100% (this corresponds to the value of 1) and the total value can change over time. For example, if the given resource becomes more expensive $\mu_i(k)$ will increase to 120% (this corresponds to the value of 1.2) and will decrease in the same way in case of a discount. It is then up to the designer to decide whether the initial value should be 100% or whether it should always be updated after a given change. The index of -1 is given for the $\mu_{c_i}(k)$ because the price function $C_i(k)$ is also considered in its inverse form. SOC(t) indicates the state of the charge of the given battery system, i.e. the power it can supply to the network. For simplicity, we will further consider $\mu_{b_i} = 1^T$. Function $\Upsilon()$ represents BESS charging or discharging decision function. Finally, w_{ij} represents the consensus weight and $\gamma_i(k)$ is the step size. Its value is discussed in the following paragraphs.

For easier generalization, it is appropriate to note that the index i now represents all types of power plants (i.e. classical and BESS). It is convenient to choose the shape for calculating w_{ij} as:

$$w_{ij} = \begin{cases} \frac{1}{d_i^{in}+1}, & j \in \mathcal{N}_i^{in} \bigcup i, \\ 0. & \text{otherwise.} \end{cases} \tag{9}$$

An important feature is each generator $i \in N_{c,b}$ need only the local information d_i^{in} to create the whole matrix W, where d_i^{in} represents the number of incoming edges. The classical generator gradient $\Phi_i(\lambda_{c_i})$ can be calculated as:

$$\nabla\Phi_i(\lambda_{c_i}) = x_{c_i}(\lambda_{c_i}) - D_i.$$

Value of $\nabla\Phi_i$ is uniformly bounded by the sum of the max. power generation x_{c_i} and the local demand D_i for i-th agent:

$$\nabla\Phi_i(\lambda_{c_i})| = |x_{c_i}(\lambda_{c_i}) - D_i| \leq \max_{i\in N} x_{c_i}^{max} + \max_{i\in N} D_i.$$

The variant for BESS can be described in a similar way, when $x_{c_i}(\lambda_{c_i})$ is replaced by the actual power provided by BESS x_{b_i}, which is based on SOC(t).

Remark 2: The virtual local demand D_i has no physical meaning. It is considered here mainly because of the possibility to design a distributed version of the algorithm. The local demand D_i can be chosen randomly among all agents. The only condition (2) that must be met is that the sum of all D_i must be equal to the total required load D.

Furthermore, the functional argument of φ in equation (7) $\nabla C_{c_i}^{-1}(\lambda_{c_i}(k+1))$ may not have a closed form solution for a general convex price function [12]. For brevity $C_{c_i}^{-1}(\lambda_{c_i}(k))$ be denoted by Γ. Next, for $\varphi_i\big(\nabla\Gamma(k+1)\big)$ the following applies:

$$\varphi_{c_i}(\Gamma) = \begin{cases} x_{c_i}^{max}, & \text{if} \quad \Gamma > x_{c_i}^{max}, \\ \nabla\Gamma, & \text{if} \quad x_{c_i}^{min} \leq \Gamma \leq x_{c_i}^{max}, \\ x_{c_i}^{min}, & \text{if} \quad \Gamma < x_{c_i}^{min}. \end{cases}$$

If the C_{c_i} is considered in the form of quadratic price function (3) then $\nabla C_{c_i}^{-1}(\lambda_{c_i})$ has the following form:

$$\nabla C_{c_i}^{-1}(\lambda_{c_i}) = \beta_{c_i}\lambda_{c_i} + \alpha_{c_i},$$

where α_{c_i} and β_{c_i} are parameters of the i-th classical generator. Similarly BESS can also only deliver the maximum amount of energy and can be limited by φ_{b_i}. This is mainly limited by the parameters $P_{\text{BESS}}^{ch_{\max}}$ and $P_{\text{BESS}}^{dis_{\max}}$ for charging and discharging the BESS, respectively and also for the lower and upper capacity limits. A control signal indicating when to charge or discharge the BESS can also be included here. For simplicity, it will be assumed here that the BESS will consistently provide these maximum values. However, this may not be the case on a real device. This value may vary if it is in the interval between 0 and these maximum values as shown in (5).

The graph $\mathcal{G}$ is used to described the network topology. The graph is assumed to be strongly connected and can be described in matrix form as in-degree adjacency matrix, denoted as W.

The step size $\gamma_i(k)$ has to be noted, for all $i \in N_{c,b}$ and $k \geq 0$ the uncoordinated step size has a expression [12]:

$$\gamma_i(k) = \frac{1}{k+1} + a_i(k) > 0, \tag{10}$$

where uncoordinated term $a_i(k)$ satisfies:

$$|a_i(k)| \leq a(k), \quad \text{where} \quad \sum_{k=0}^{\infty} a(k) < \infty.$$

Premise (10) for $\gamma_i(k)$ gives each unit with flexibility in choosing the decay $\gamma_i(k)$ thanks to the $a_i(k)$. For example, if $a_i(k)$ is chosen as:

$$a_i(k) = \frac{M_i}{(k+1)^{c_i}} - \frac{1}{k+1}, \tag{11}$$

where $M_i > 0$ and $c_i \in \langle 0.5, 1 \rangle$ represents tunable parameters of $a_i(k)$. If $a_i(k)$ has the form given in (11) then the step size $\gamma_i(k)$ can be calculated by (12). The form (12) will be used in the following examples.

$$\gamma_i(k) = \frac{M_i}{(k+1)^{c_i}} \tag{12}$$

IV. Simulation Examples

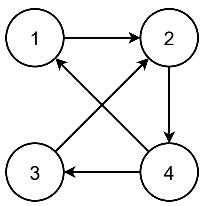
The algorithm is presented using (by means of) three examples. Three types of generators and one BESS will be considered. They are **Type I** - coal-fired steam unit, **Type II** - oil-fired steam unit and **Type III** - oil-fired steam unit with various parameters. The generator parameters were taken from [2]. Their values are noted in the table III. For BESS, a more complex model described in II-A2 will be considered.

TABLE III

Generators parameters			
Generator type	**Type I**	**Type II**	**Type III**
Range $[MW]$	[150, 600]	[100, 400]	[50, 200]
α [\$ / MW^2h]	-2535.2	-2023.6	-826.8
β [\$ / MWh]	352.1	257.7	103.7
γ [\$ / h]	-8616.8	-7613	-3126.7

A. Variable fuel price

The network topology considered for this example is shown in Figure 2. Vertices 1 and 2 are **Type I** generators. Vertex 3 is **Type II** generator and vertex 4 is **Type III** generator. The weight matrix W is set according to the (9).



$$W = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

Fig. 2: 4 agents network topology

The desired load will be set to $D = 1000$ MW. The individual D_i are set randomly, with the only restriction that their sum must be equal to D. The initialization of individual variables is carried out according to the Table IV. Parameter γ represents the step size and its value was set based on the equation (12).

This example focuses on the effect of fuel price changes on the course of the algorithm. The whole example was simulated for 200 steps (seconds)with a total of two changes in the values of the price function. The initial value μ and its changes are considered as:

$$\mu_0 = [1, 1, 1, 1],$$
$$\mu_{70} = [1.2, 1.1, 1.05, 1],$$
$$\mu_{140} = [0.95, 0.9, 0.8, 0.8].$$

TABLE IV

Initialization for the algorithm				
Variable	$i = 1$	$i = 2$	$i = 3$	$i = 4$
x(0) $[MW]$	150	150	100	50
y(0) $[MW]$	300	300	150	200
λ(0)[\$/MWh]	7.6262	7.6262	8.2390	8.4552
D(0) $[MW]$	450	450	250	250
M(0)	0.8	0.8	0.8	0.8
c(0)	0.85	0.85	0.85	0.85
γ(0)	0.8	0.8	0.8	0.8

The simulation results for the values of $\lambda_i(t)$ and $x_i(t)$ are shown in Figure 3. The algorithm responds correctly to changes in fuel price. The plots show that the total incremental cost λ increases when the fuel price is higher in the range from $t = 71$s to $t = 140$s. In the range from $t = 0$s to 70s, its value was equal to 8.357 \$/MWh. Next, from $t = 71$s to 140s its value was equal to 8.483 \$/MWh and in the range from $t = 141$s to 200s its value was equal to 8.483 \$/MWh.

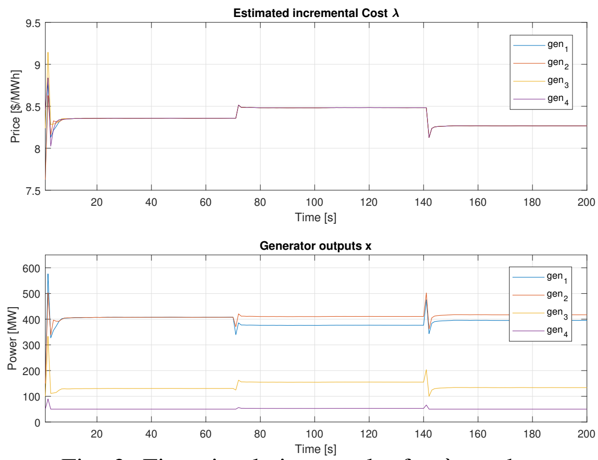


Fig. 3: First simulation results for λ_i and x_i

Remark 3: The results were compared for distinctive values with the results from the Lagrangian multipliers and the Lambda-iteration method. They were identical and therefore it can be assumed that the presented distributed algorithm gives the same results as classical centralized methods.

B. Battery system representation

The network topology considered for this example is shown in Figure 4. The parameters of the generators are the same as in the previous example. The weight matrix W is set according to the (9) again. And its value is as follows, where the green high-lighted line represents the BESS. The parameters for the BESS were established in Table II, with the only difference that the maximum capacity SOC_U is considered to be 100MW. Minimum capacity SOC_L will be set as 10MW.

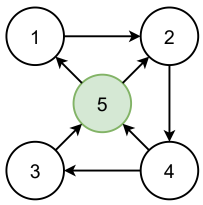


Fig. 4: 5 agents network topology

$$W = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

The desired load will be again set to $D = 1000$ MW. The individual D_i are set randomly and with the same restriction like it was in the previous example. The initialization of each variable is noted in the Table V. Where term $i = 5$ corresponds to BESS.

TABLE V

Initialization for the algorithm					
Variable	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
x(0) $[MW]$	150	150	100	50	0
y(0) $[MW]$	300	300	150	200	25
λ(0)[\$/MWh]	7.6262	7.6262	8.2390	8.4552	1
D(0) $[MW]$	450	450	250	250	25
M(0)	0.8	0.8	0.8	0.8	0.8
c(0)	0.85	0.85	0.85	0.85	0.85
$\gamma(0)$	0.8	0.8	0.8	0.8	0.8

This example is considered on the time horizon of one day, which was divided into seconds for simulation purposes. It therefore consists of a total of 86400 steps (seconds). This example will shows the charging and discharging of the BESS and its effect on the resulting incremental cost for the entire system. At the beginning, the BESS will be charged. Subsequently, in step $t = 40000$, the BESS will be switched to discharge and capacity will be provided to the network until it discharges to the minimum capacity value SOC_L.

The simulation results for the values of $\lambda_i(t)$, $x_i(t)$ and BESS SOC(t) are shown in Figure 5.

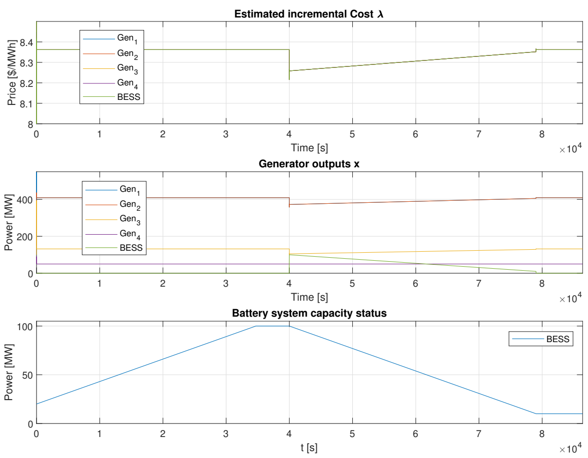


Fig. 5: Second simulation results for λ_i, x_i and BESS capacity

It is clear from the results that providing the power stored in the BESS, which has a lower cost function, leads to a reduction in the total incremental cost λ for the whole network. In the range for $t = 0$s to 70s the total incremental cost λ value was equal to 8.357 \$/MWh. Next, in the range for $t = 71$s to 140s, its value was equal to 8.483 \$/MWh and in the range for $t = 141$s to 200s its value was equal to 8.483 \$/MWh. At the same time, the state of charge of the BESS can be seen on the bottom graph 5.

C. Response of the battery system to a change in the fuel cost

In this case, the same topology and initialization as in IV-B are considered. The generator parameters are again listed in Table V. The same applies to the BESS parameters listed in Table II and the capacity limitation. The total required load is again set to $D = 1000$ MW.

This example focuses on the response of BESS to a change in fuel price. There were a total of six fuel price changes over a period of 86400 steps. This can be transferred to a time horizon of one day and can be interpreted as seconds. Also in this example, for simplicity, the fuel price for the BESS will be considered unchanged throughout the simulation. The initial value μ and its changes are considered as:

$$\begin{aligned} \mu_0 &= [0.8, 0, 7, 1, 1, 1], \\ \mu_{10000} &= [1.2, 1.2, 1.15, 1.15, 1], \\ \mu_{20000} &= [1, 1, 1, 1, 1], \\ \mu_{30000} &= [0.75, 0.75, 0.8, 0.8, 1], \\ \mu_{50000} &= [1, 1, 1, 1, 1], \\ \mu_{60000} &= [1.3, 1.3, 1.2, 1.15, 1], \\ \mu_{750000} &= [1, 1, 1, 1, 1]. \end{aligned}$$

The logic for controlling the BESS function $\Upsilon()$ was chosen so that when the fuel price drops to 80% of the original price or less, the BESS start charging. Conversely, if the price function rises above 120% of the original price, the BESS will discharge. In between, BESS will not take any action. The simulation results for the values of $\lambda_i(t)$, $x_i(t)$, BESS SOC(t) and fuel cost $\mu(t)$ are shown in Figure 6.

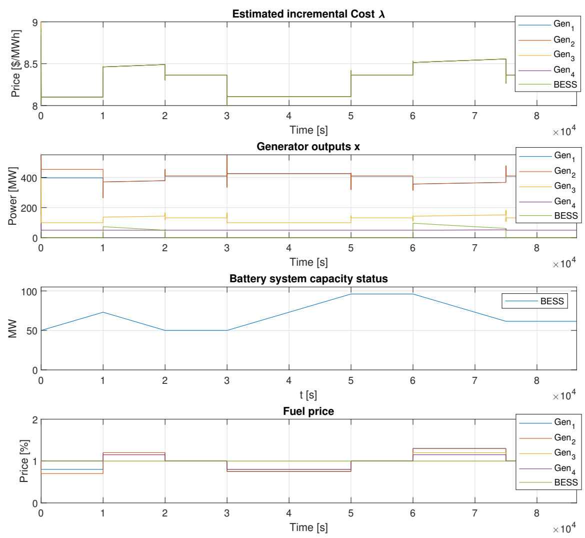


Fig. 6: Third simulation results for λ_i, x_i, BESS capacity a fuel cost

Figure number 6 shows the reactions of the algorithm to all fuel price changes. In the interval from $t = 0$s to $t = 10000$s, the BESS was charged. Furthermore, the BESS was charged in the interval $t = 30000$s to $t = 50000$s. On the contrary, after the fuel price fell below the defined limit, the energy was returned to the grid. So the discharge continued in the interval $t = 10000$s to $t = 20000$s and then again in the interval $t = 50000$s to $t = 75000$s. During the interval $t = 20000$s to $t = 30000$s there was no discharging or charging of the BESS either. This situation was repeated from $t = 75000$s until the end of the simulation. The upper graph in the Figure 6 shows the evolution of $\lambda_i(t)$ throughout the simulation and the influence of the fuel price on its resulting value.

Remark 4: The BESS control logic was considered to be very simple and was only intended to document the possible integration of BESS into the distributed algorithm. In the real world, this control logic will need to be much more complex.

V. Future works

A first possible improvement would be to include a representation of renewable resources in the algorithm [16]. Specifically this would be wind and solar power plants. Thanks to this, the formulation of the task would also be expanded to include the price functions of these resources. However, to maintain the stability of the energy network their inclusion should not exceed $30 - 40\%$ of the total required power D. At this point, it is offered to store excess energy directly in battery systems, of course depending of course on the current weather or its forecast. If one wanted to control battery systems on a given time horizon, it would be possible to use methods based on the Model Predictive Control theory or other Rolling Horizon Strategy [21, 31, 19].

Another point could be to include more uncertainties in the calculations and network representation. In the previous work, the authors worked with time-varying traffic delays, gradient calculation noise, power line losses and representation of drop-off packet communication [29]. The aim could therefore be to develop a robust algorithm incorporating the representation of the BESS.

Another topic can be to consider emission production in the model for the EDP solution [32]. Methods for solving EDP based on this point of view represent optimization algorithms whose goal is to distribute the total demand among all generators in the network so that the resulting price is minimal and at the same time the emissions produced are minimal.

VI. Conclusion

The paper deals with a gradient algorithm for a distributed way of solving EDP which simultaneously considers the integration of the BESS and variable fuel prices within the entire iterative algorithm. Two models were considered for the BESS, but it shouldn't be a problem to replace them with an arbitrarily complex model. The algorithm provides possibilities for further extension which are described in Chapter V.

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